Equilibrium states of two-dimensional turbulence: An experimental study

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Equilibrium states of vortex arrays, excited by electromagnetic forces, are studied experimentally in thin, stably stratified, fluid layers. Several characteristics (the conservation of the maximum vorticity and the structure of the final state) are found to be in conflict with statistical theory, while better agreement is obtained with a recent approach in which the final state is controlled by the surviving structures of the decay phase.

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The problem of determining the equilibrium state of two-dimensional turbulence has been addressed a long time ago and is still controversial. The statistical approach, which was initiated by Onsager [1], has been widely discussed over the past 20 years [2]. Recently, this theory was extended to the case of continuous fields [3]. This approach implies the existence of well defined equilibrium states, controlled by the global invariants of the system and thus independent of the precise structure of the initial flow conditions. If this approach is relevant to two-dimensional turbulence, the powerful tools of statistical mechanics may possibly be used for attacking nonequilibrium situations such as forced turbulence [3]. The issue of ergodicity has been raised by several authors [4]: In two-dimensional turbulence, because of the absence of an efficient process for mixing the vorticity patches and the observed tendency to form robust, long-lived, coherent structures, it is not clear whether the flow can be treated as an ergodic system. An alternative approach was proposed recently [5] (originally for large population of vortices), suggesting that the end state is the ultimate stage of a self-similar evolution governing the decay phase. As the largest structures are formed, the system eventually ceases to evolve or evolves in a low-dimensional dynamical space. The two approaches differ in many respects. In the scaling theory, the final state is selected by some self-similar process at work in the decay phase, while in the statistical theory it is selected by a maximum entropy condition. In scaling theory, the maximum vorticity is conserved and the vorticity patches occupy only a fraction of the available space, two characteristics that in general differ from the statistical theory. There are also other approaches to the problem of equilibrium, including "selective decay" theory [6], which assumes that the equilibrium states coincide with an enstrophy minimum. There is no strong justification for this hypothesis, except that enstrophy must decrease with time in a freely evolving system so that if a steady state is to be reached, it must correspond to some minimum.

On the experimental side, the situation is somewhat puzzling: The results obtained in a plasma experiment [7]—assumed to be isomorphic to a fluid system—tend to support selective decay theory, while those obtained in mercury [8] tend to support statistical theory. In this Brief Report we examine a different class of initial conditions, characterized by the presence of several structures in the system. It will appear that in this case, the predictions of some of the various approaches strongly differ from each other, even on a qualitative level, so that sharp distinctions can be made.

The situations that we consider are arrays of vortices confined in square boxes. The system is similar to a previous experiment [9] (which actually used nonstratified fluids) and we give here only a brief description of the experimental setup. The flow is produced in thin layers of thickness b equal to 6 mm, consisting of two layers of salt water of different concentrations with the same thickness. To produce the initial flow, an electric current I is driven horizontally, from one side of the cell to the other, and just below the flow, arrays of permanent magnets are formed. The interaction of the periodic magnetic field with the electric current produces a system of recirculating flows whose structure is imposed by the arrangement of the magnets. In the experiments described here, regular arrays of four, nine, and sixteen counterrotating vortices and irregular arrays to ten vortices are considered. The vortex dimensions are 16×16 mm^2 and the initial vorticity profile is roughly Gaussian. The flow is visualized by particles, several tens of micrometers in size, slightly lighter than the fluid; they are deposited on the free surface and visualized from above. The images are sent to a video recorder and further analyzed. To compute the instantaneous velocity, we use a technique already used in a previous work [9] and based on the calculation of correlations: the system is discretized into 40×40 squares and maxima of temporal correlations of the light intensity diffused by the particles are calculated; the resulting velocity field is fitted by spline functions. The calculations of the vorticity field and the stream function are done in the direct space. The accuracy of the measurement of the velocity field can be estimated to a few percent and that of the vorticity to about $\pm10\%$. Related quantities, such as the energy, the enstrophy, the maximum vorticity, and the local divergence, are calculated.

To obtain an equilibrium state, we first impose a steady value of I at $t=-\tau$, after which we quench it at $t=0$.

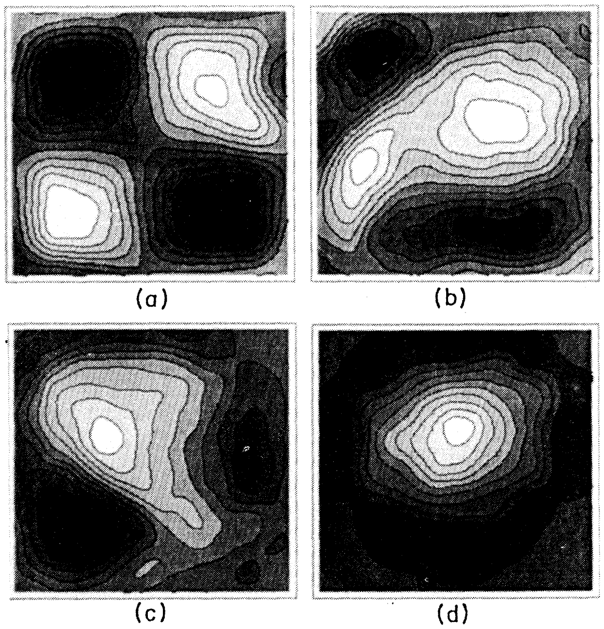

FIG. 1. Evolution of the vorticity field for a system of four counterrotating vortices, enclosed in a square box: (a) $t=0$ s, (b) $t=2.3$ s, (c) $t=8.2$ s, and (d) $t=28$ s.

During the relaxation process, because of the fraction exerted by the bottom wall on the fluid layer, the total kinetic energy decreases exponentially, with a time constant of typically 30 s. We typically follow the system during ten turnover times before most of the energy is dissipated; the stationarity of the end state is controlled by inspecting several quantities, such as the flow pattern, the enstrophy, or the ω-ψ plot (where ω and Ψ are, respectively, the vorticity and the stream function). The issue of the two dimensionality of the flow is crucial and we perform several tests to discuss it. In our system, three dimensionality arises because of three-dimensional flows present within the fluid layer and the deformation of the fluid interface. Concerning the interface deformations, we estimate them as a few percent of the total height in the decay regime. This is consistent with the typical values of the Froude number that we work with (smaller than 0.5). Concerning the three-dimensional flows, we measure the horizontal divergence of the velocity field, which is found to be 5% of the maximum vorticity in all the experiments; this indicates that they are weak. Additional information is provided by the inspection of elementary situations, such as the single vortex, the dipole, the merging two like-sign vortices, and the dipole collision, which we compare to two-dimensional calculations. In all cases, we find excellent consistency between our observations and the theoretical expections. We thus may consider that our system is essentially two dimensional.

Figures 1(a)–1(d) show the evolution of the isovorticity lines for a system of four counterrotating vortices, produced initially by an electric current of 2 A, imposed during a time $\tau=1$ s, in a stepwise stratified layer of 6 mm. After the current is quenched, two like-sign vortices merge so as to form, a few seconds later, a central vortex [Fig. 1(d)]. The merging is asymmetric: During this process, the core of one of the two vortices is preserved, while the other is stretched around it so as to form an annular vorticity layer. The walls themselves generate ribbons that mix with the outer layer, without, apparently, perturbing the core of the flow; such ribbons are responsible for the slight increase of the mean velocity of the system (10% of the maximum value) during the decay phase. The final state, in this case, is thus an asymmetric annular structure for which the active zone of the vorticity field occupies the majority of the box area, but not its totality. The free decay of a system of nine vortices show a more complex sequence of mergings: The first event is the merging of the four vortices located around the central one. During this process, the central vortex is expelled at the periphery and at the end we obtain a structure somewhat close to a dipolar one (see Fig. 2). In this case, it is more evident that the active structure of the final state occupies only a fraction of the box area, leaving the other part filled with patches of weak vorticity. Finally, the disordered configuration with ten vortices also shows that the decay consists of a succession of mergings and one clearly obtains, for the final state, a dipolar structure, which occupies an area less than 30% of that of the box (see Fig. 3). A similar situation is observed for the sixteen-vortex system.

An interesting quantity is the maximum vorticity ζ_{max}, which, according to "scaling theory," is conserved during

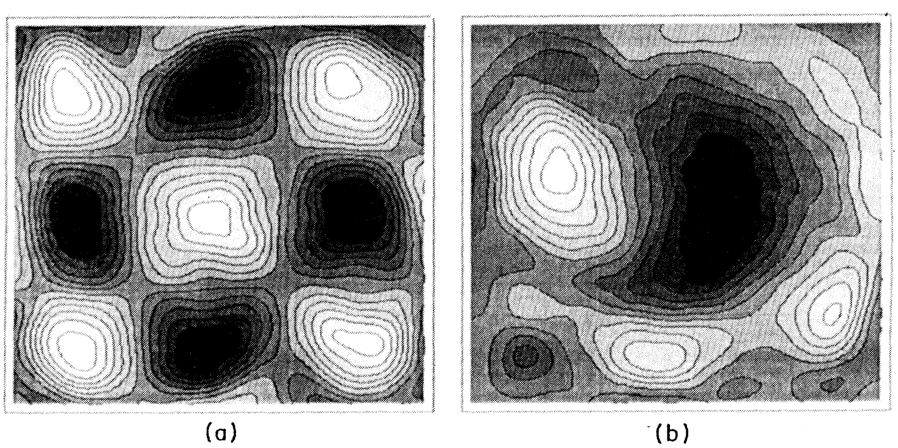

FIG. 2. (a) Initial and (b) final states for a system including initially nine counterrotating vortices arranged on a regular lattice.

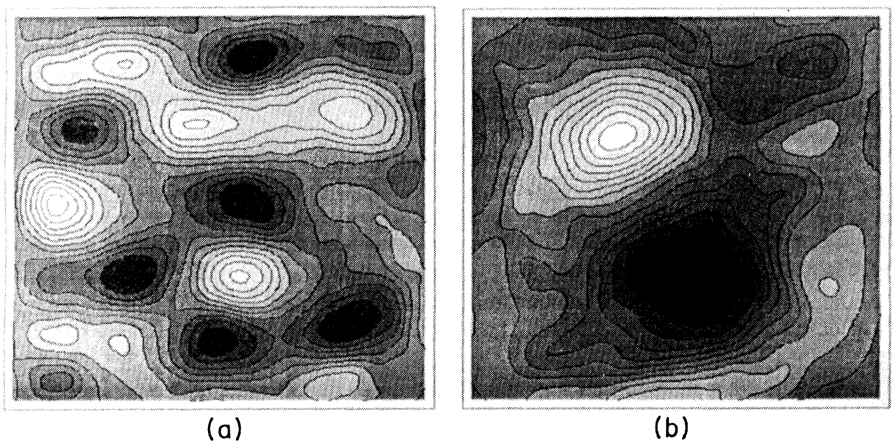

FIG. 3. (a) Initial and (b) final states for a system including initially ten counterrotating vortices randomly spaced.

the decay phase, while it may decrease for the two other approaches. The evolution of this quantity (renormalized by $\sqrt{E}$, where E is the total kinetic energy [10]) is shown in Fig. 4. One finds that $\zeta_{max}/\sqrt{E}$ is constant in all cases. According to the usual interpretations, and in agreement with the direct observation of the vorticity field, one can infer that the core of the strongest vortex is preserved during the mergings and therefore complete mixing of the vorticity levels is not achieved during the decay phase, at least for the available time of observation. Concerning the end states themselves, the corresponding ω-Ψ plots are shown in Fig. 5. We find different situations: In the four-vortex system, one can tentatively consider that we are close to an equilibrium state, since the scatter on the plot is fairly small (close to the origin, there is some scatter, but this is due to the ribbons induced by the walls). In the other cases, this is less clear; in particular, the case of the initially disordered system [Fig. 3(c)] is certainly nonstationary [the fact that the state of Fig. 3(c) is nonstationary is evident].

These observations are in good correspondence with scaling theory even when the initial state is not a large population of vortices. Some characteristics are the

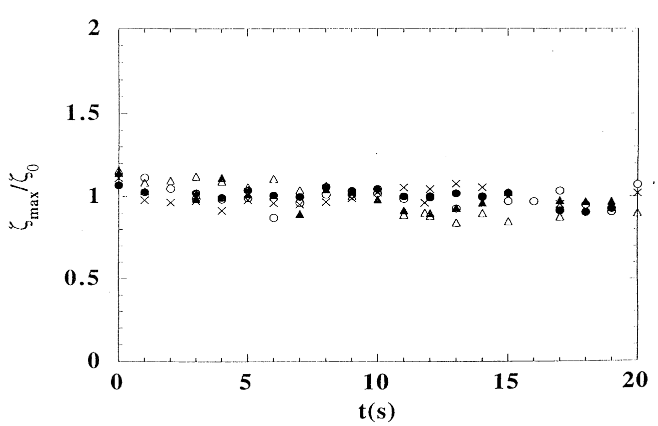

FIG. 4. Evolution of the renormalized maximum vorticity for four different cases: ▲, four vortices; △, nine vortices; ●, ten vortices, randomly spaced; ×, sixteen vortices. The expression of ζ_0 is $\zeta_{max}(0)(E(0)/E)^{1/2}$.

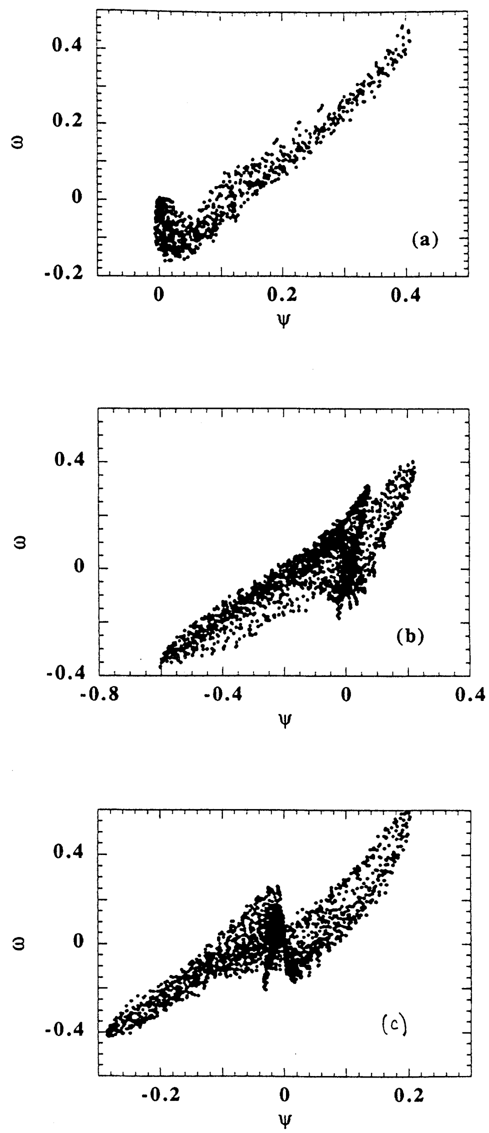

FIG. 5. Scatter plots for the end states, obtained after 28 s, in the cases corresponding to Figs. 1–3: (a) four vortices, (b) nine vortices, (c) ten vortices, randomly spaced.

same: the conservation of the vorticity extremum of the strongest structures and the decrease of the area occupied by active vorticity patches, two features that are present in both this approach and our experiments. The fact the final state of flow is essentially controlled by the surviving structures is in good agreement with our observations; this is particularly clear for the disordered system. In contrast, there are significant differences between our results and the other theories. To discuss the relation with statistical theory, one should decompose the initial velocity field into a set of N patches of amplitude a_i and $-a_i$ (i varying from 1 to N). We consider here a simpler approach by introducing only two vorticity levels $+a$ and $-a$, respectively, for defining the initial state. With this simplification, the equilibrium state is determined by the relation

$$\omega = a \tanh(-\alpha + \beta a \Psi) \quad (1)$$

in which α and β are constants. This equation has been computed for some (positive) values of these constants [11]. One finds, in the physical space, a structure consisting of a central, symmetric vortex, surrounded by a vortex layer, and occupying the totality of the available area. One could say that expression (1) is not in conflict with the curve of Fig. 3(a), corresponding to the four-vortex system; the corresponding structure, in the physical space, is not so distinct from the expected one (the main difference is the asymmetry observed in the experiment). Actually, one can show that there is an incompatibility between the fact that ω is roughly a linear function of Ψ and the conservation of the maximum of the vorticity. In the case of larger vortex populations, the structure that is obtained—a dipole occupying a reduced fraction of the box area—is clearly different from the predictions. The only way of reconciling theory and experiment in this case would be to assume that the dipole will further evolve towards a steady state described by (1). Although we have no direct experimental evidence that this is not the case, one can say that this is improbable. There is no mechanism, in the bulk, that would destroy the dipolar structure; concerning the walls, we have checked that they can be roughly assimilated to mirrors, so that one can hardly see any mechanism that would reorganize the vortex couple into an annular symmetric structure. By studying the case of a dipole enclosed in a box, we have observed first a propagation and then a separation of the two vortices as the dipole reaches a wall and no tendency to form an annular structure. Therefore there are strong indications that the maximum entropy state is unlikely to be reached and that statistical theory does not apply for this case. Similar conclusions can be drawn concerning the selective decay theory. According to this theory, for the equilibrium state, the system selects the gravest mode compatible with the boundary conditions; this would give a symmetric annular structure in our case and not an isolated dipole.

We therefore are led to conclude that, for the situations that we have investigated, the observed final states of flow display characteristics in conflict with both statistical and selective decay theories and are in better agreement with the approach of Ref. [5], originally proposed for large populations of vortices. The relevance of scaling theory, which has already been shown in numerical experiments, turns out to apply also to some real flows. The picture that now seems to emerge, on the experimental side, is that when many structures are initially present in the system, vortices merge at random and we end up with a dipole that fully controls the final state; the system does not reach a maximum entropy state in this case. Now, when the vorticity patches are initially organized into a single structure, the situation is less clear and somewhat puzzling since contradictory results have been reported for this situation. Finally, the situation of the experiment is not so far from that found in the numerics [12].

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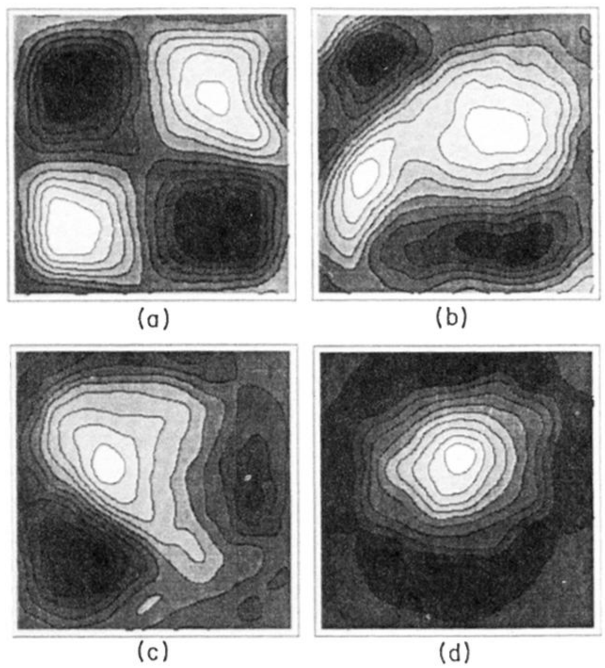

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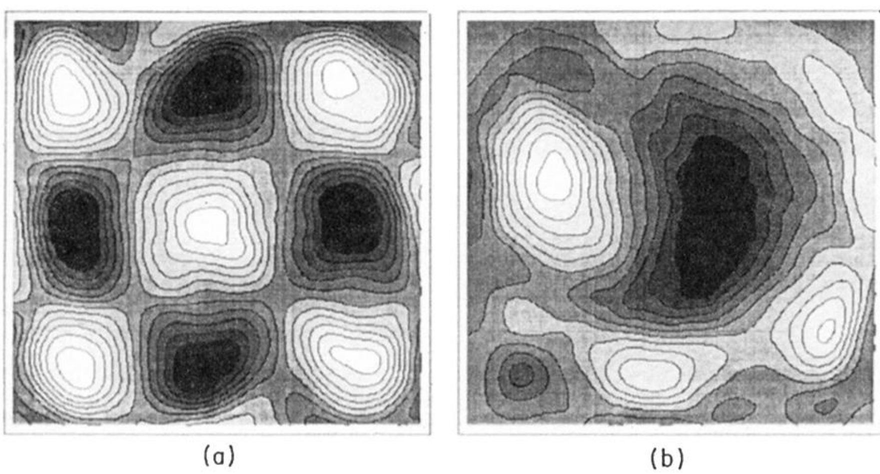

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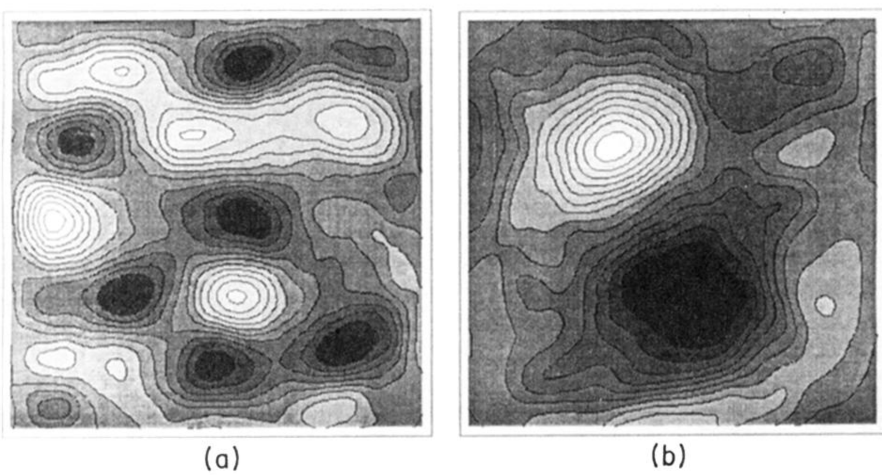

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